Maths Department	Math 241	7 pages
14/7/1438H	Exam 1	90 minutes [25 marks]

Name:		Student ID:	
Student Section:		Serial number:	

Marks: $\dfrac{\quad}{25}$

Q1: Mark True (T) or False (F) and justify your answers: [5 marks]

(1) [ ] If one row in an echelon form of an augmented matrix is $\begin{bmatrix} 0 & 0 & 0 & 5 & 0 \end{bmatrix}$, then the associated linear system is inconsistent.

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(2) [ ] If A is $n \times n$ matrix, then $A - A^T$ is skew-symmetric.

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(3) [ ] If $A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$, then $A^2 = \begin{bmatrix} 4 & 1 \\ 9 & 0 \end{bmatrix}$

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(4) [ ] If AA^T is singular matrix, then A is also singular.

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(5) [ ] If A and B are $n \times n$ matrices such that A is an invertible matrix, then for any matrix B; $|A^{-1}BA| = |B|$.

Q2: Fill in the blanks: [5 marks]

(1) If A is 3×3 matrix such that $|A| = 9$, then $|3A^{-1}| = \ldots$

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(2) If $A \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $A = \begin{bmatrix} \ldots & \ldots \\ \ldots & \ldots \end{bmatrix}$

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(3) If $\begin{vmatrix} \mathrm{a}_{11} & 2 & 1 \\ 0 & \mathrm{a}_{22} & -1 \\ 0 & 0 & 5 \end{vmatrix} = 15,\ \textit{then}\ \mathrm{a}_{11} = \ldots$ and $a_{22} = \ldots$

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(4) If $A = [a_{ij}]$ is $n \times n$ skew-symmetric matrix, then $a_{ii} = \ldots\ \forall i = 1, 2, \ldots, n$

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(5) The system $\begin{aligned} x + y - 2z &= 1 \\ 3x + 3y - 6z &= 2 \end{aligned}$ has ...... solution(s)

(6) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 3 & -2 & -20 \end{bmatrix}$, then $Trace(A) = \dots$

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(7) $A = \begin{bmatrix} k-1 & 2 \\ 4 & k+1 \end{bmatrix}$ is singular if $k= \dots$

Q3: For what values of k the following system has: [2.5 marks]

(a) No solution.

(b) An infinite number of solutions.

(c) Exactly one solution.

$$x + 2y - z = 3$$
$$-x - y + z = 2$$
$$-x + y + z = k$$

Q4: Let A be an investable $n \times n$ matrix, prove that: [4.5 marks]

(a) $AB = AC \Rightarrow B = C$

(b) $|A^{-1}| = \dfrac{1}{|A|}$

(c) If A is orthogonal, then $|A| = \pm 1$

Q5: Verify that the equation [2.5 marks]

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

Q6: Use Cramer's rule to solve the system, [2.5 marks]

$$5x + 4y = 2$$

$$-x + y = -22$$

Q7: Find A^{-1} by using the adjoint matrix, where [3 marks]

$$A = \begin{bmatrix} 4 & -2 & 3 \\ 2 & 2 & 5 \\ 8 & -5 & -2 \end{bmatrix}$$